THE UNIVERSITY OF HONG KONG

Institute of Mathematical Research
Department of Mathematics

COLLOQUIUM

On Brent-Salamin Algorithm for π

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Date:	July 17, 2013 (Wednesday)
Time:	3:00 – 4:30pm
Venue:	Room 210, Run Run Shaw Bldg., HKU

Abstract

The Gauss' "Arithmetic-Geometric Mean" two-term recurrence is defined by

(1) $$a_{n+1} = \frac{a_n + b_n}{2}$$

and

(2) $$b_{n+1} = \sqrt{a_n b_n},$$

where $0 < b_0 \leq a_0$. In 1976, R. Brent and E. Salamin discovered independently an efficient algorithm for computing π using Gauss' "AGM" recurrence. The Brent-Salamin algorithm states that if $a_0 = 1$, $b_0 = 1/\sqrt{2}$ and a_n and b_n are given by (1) and (2), and

$$\pi_n = \frac{2a_n^2}{1 - \sum_{j=0}^{n} 2^j \left(a_j^2 - b_j^2\right)},$$

then π_n increases monotonically to π. In this talk, we will give a proof of the Brent-Salamin algorithm and present some new algorithms. One of our algorithms is the following: Let $\breve{a}_0 = 1$ and $\breve{b}_0 = \frac{1}{\sqrt{2}}$. Let

$$\breve{a}_{n+1} = \frac{\breve{a}_n + 3\breve{b}_n}{4}, \breve{b}_{n+1} = \sqrt{\frac{\breve{b}_n(\breve{a}_n + \breve{b}_n)}{2}}$$

and

$$\breve{\pi}_N = \frac{2\sqrt{2}\breve{a}_N}{1 - \sum_{j=0}^{N} 2^j (\breve{a}_j - \breve{b}_j)}.$$

Then $\breve{\pi}_N \to \pi$.

All are welcome